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FORCED CONVECTIVE HEAT TRANSFER IN A STRAIGHT PIPE ROTATING AROUND A PARALLEL AXIS

(2ND REPORT, TURBULENT REGION)

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Abstract—Turbulent heat transfer in a pipe rotating around a parallel axis with large angular velocity is studied theoretically by assuming an effective secondary flow due to density difference in a centrifugal field. Flow and temperature fields, fully developed under the condition of constant wall temperature gradient, are analyzed by dividing them into a flow core region and a thin boundary layer along the wall. The analysis consists of two parts in this paper. One is common to different secondary flow problems, and the other in which the body force terms appear explicitly is particular for the present problem. The formulae for the resistance coefficient λ and the Nusselt number Nu are obtained in terms of Re and Γ (Re = Reynolds number, Γ = inertia force/body force). It is shown that the increases in the values of λ and Nu due to secondary flow can be expressed in terms of the parameter $Re/\Gamma^{m/2}$ ($m = 1$ for laminar flow, $m = 4$ or 5 for turbulent flow) which is found to be a general parameter for various secondary flow problems.

The effect of Cariolis force due to secondary flow is shown to be negligibly small.

NOMENCLATURE

A,	w_1 at the pipe axis;
A',	g_1 at the pipe axis;
a,	radius of the pipe;
C,	$\equiv -(\partial P/\partial z)$;
c_p,	specific heat of fluid at constant pressure;
D,	dimensionless velocity of the secondary flow in the flow core;
G,	$\equiv T_w - T$;
Gr,	Grashof number, $\equiv R\omega^2\gamma(\tau a)(2a)^3/\nu^2$;
g,	$\equiv G/\tau a$;
g_m,	$\equiv (T_w - T_m)/\tau a$;
J,	$\equiv 2a^2\omega/\nu$;
k,	thermal conductivity of fluid;
M,	(Coriolis force due to secondary flow)/(body force);
m,	constant defined by equation (15) or equation (23);
Nu,	Nusselt number, $\equiv [2aQ_{wm}/k(T_w - T_m)]$;
Nu_0,	Nu with no secondary flow;
n,	constant defined by equation (26);
P,	$\equiv (a^2/\nu^2)(p/\rho)$;
Pr,	Prandtl number, $\equiv \rho c_p \nu/k$;
p,	pressure;
Q_η, Q_ψ,	heat flux in the fluid;
Q_w,	heat flux at the wall;
q_η,	$\equiv Q_\eta/k\tau$;
q_ψ,	$\equiv Q_\psi/k\tau$;
q_w,	$\equiv Q_w/k\tau$;
R,	radius of rotation of the pipe axis;
Ra,	Rayleigh number, $\equiv GrPr$;
Re,	Reynolds number, $\equiv 2aW_m/\nu$;
r,	radial co-ordinate in a cross section;
T,	temperature;
T_m,	mixed mean fluid temperature;
T_w,	wall temperature;
U,	radial component of velocity;

u, $\equiv Ua/\nu$;
V, circumferential component of velocity;
v, $\equiv Va/\nu$;
v^{*2}, circumferential component of $\breve{w}^{*2}$;
W, axial component of velocity;
w, $\equiv Wa/\nu$;
W_m, mean velocity;
w^{*2}, axial component of $\breve{w}^{*2}$;
$\breve{w}$, resultant velocity formed by v and $\breve{w}$;
$\breve{w}^{*2}$, $\equiv (a^2/\nu^2)(\tau_w/\rho)$;
X, body force;
Z, axial co-ordinate;
z, $\equiv Z/a$.

Greek symbols

α, proportional coefficient in the formula for λ_0, equation (15);
α_r, proportional coefficient in the formula for λ, equation (71);
β, proportional coefficient in the formula for Nu_0, equation (23);
Γ, (inertia force)/(body force);
γ, coefficient of volumetric expansion of fluid;
Δ, $\equiv \psi - \psi'$;
Δ_x, coefficient of the correction term (subscript x denotes the correction term for the quantity x);
δ, dimensionless thickness of the boundary layer;
ζ, correction coefficient due to Coriolis force;
η, $\equiv r/a$;
κ, exponent of Pr in the formula for Nu_0, equation (23);
λ, resistance coefficient, $\equiv [(-\partial p/\partial Z)(2a/\tfrac{1}{2}\rho W_m^2)]$;
λ_0, λ with no secondary flow;
ν, kinematic viscosity;
ξ, $\equiv 1 - \eta$;
ρ, density;
τ, temperature gradient along the pipe axis;
τ_w, shear stress at the wall;
$\tau_{z\eta}$, $\tau_{z\psi}$, dimensionless shear stresses in the direction of the pipe axis;
ψ, angular co-ordinate in a cross section whose original line ($\psi = 0$) lies on a secondary flow streamline passing through the center of a cross section;
ψ', angular co-ordinate in a cross section, the extension of whose original line ($\psi' = 0$) passes through the center of a cross section and the center of rotation;
ω, angular velocity of the pipe.

Subscripts

0, value obtained by neglecting the Coriolis force in Section 2.6;
1, value in the flow core region;
4, λ and Nu calculated by putting $m = 4$;
5, λ and Nu calculated by putting $m = 5$;
f, value at the position where v in the boundary layer becomes maximum;
m, mean value around the periphery ($\psi = -\pi \sim \pi$) or on a semi-circle $\psi = 0 \sim \pi$ (section 2.3);
δ, value at $\xi = \delta$.

INTRODUCTION

IN A PIPE rapidly rotating around a parallel axis, a fluid is subjected to a centrifugal force much larger than the natural gravity. The difference in density of the fluid due to the temperature distribution in the flow causes a secondary flow which increases remarkably the flow resistance and the heat-transfer rate. In the cooling passages in rotating hot parts of various engines and machines, we see the heat-transfer problem discussed in this paper.

Especially, recent requirements for high performance jet engines, gas turbines and electric generators demand designers to employ effective cooling technique in order to save the significant loss of material strength in high temperature environments. Since high temperature in rotating material has a great influence on material strength and life, it is very desirable to evaluate heat-transfer coefficients in the cooling passages with some accuracy.

In spite of the practical importance of the problem, the available data are insufficient for design purposes. The experimental work was done by Humphrey *et al.* [1] in the entrance region where air was introduced from the rotating axis through a passage formed by a radial rotating pipe and a bend. However, the results obtained were under the complicated effects of the different factors, and fail to give a correlation function. Morris [2] made a theoretical study for a fully developed laminar flow using a perturbation method. His results are confined to a small Reynolds number and low rotational speed region, since the secondary flow is assumed to be very small.

Mori and Nakayama [3] made an analytical study in fully developed laminar region assuming that the effect of secondary flow is predominant in the pipe except in a thin layer along the wall. The thin layer next to the wall is called the boundary layer in the sense that the boundary-layer approximation can be applied in this region, while the central part is called the flow core. The results are obtained for the practically important region of large Reynolds numbers and high rotational speeds. The increase in the flow resistance and that in the Nusselt number are shown in terms of $RaRe$, where Ra is the Rayleigh number based on the centrifugal acceleration and Re is the Reynolds number. In addition, the effect of Coriolis force due to secondary flow, though it is small, is discussed, and the correction coefficients taking this effect into account are given.

In the present paper, turbulent heat transfer in a rotating pipe is studied theoretically assuming that the effect of secondary flow is sufficiently large. The flow and temperature fields are fully developed under the condition of constant wall temperature gradient, and analyzed by dividing them into the flow core and the boundary layer. The analysis is done following the method of analysis for turbulent heat transfer in curved pipes done by Mori and Nakayama [4]. The analysis consists of the part to be applied commonly to various secondary flow problems and the one which is particular in the present problem. The change in density of the fluid is taken into account only in the terms of centrifugal force in the momentum equations, and other physical properties are regarded as constant. The fluids discussed here have Prandtl numbers of about unity and greater than unity.

1. THE ANALYSIS BY THE BOUNDARY-LAYER APPROXIMATION

1.1. *Fundamental equations*

It is pointed out in the first report on laminar flow [3] that, in most practical cases, a sufficiently large secondary flow distorts the velocity and temperature distributions in a pipe remarkably from the symmetrical profiles. The velocity and

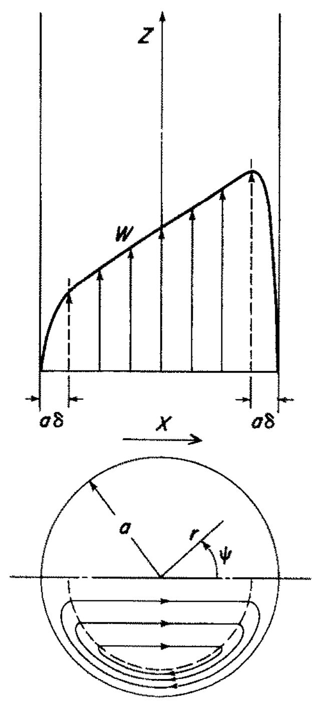

FIG. 1. System of co-ordinates.

temperature distributions observed in turbulent flow in the curved pipes [4] also imply the significant effect of secondary flow. A similar effect can reasonably be expected in turbulent flow where intense secondary flow is caused by the different body forces.

In Fig. 1, the body force X is applied in the direction indicated by the arrow. When a secondary flow is large, velocity and temperature change gradually in a pipe except in a boundary layer next to the wall where they change steeply from the values at a flow core to those at the pipe wall. The axial velocity distribution (w) is shown in Fig. 1. This feature of velocity and temperature profiles may be explained by the predominant role of the shear stress and the heat flux due to secondary flow in the flow core region. The secondary flow is assumed to be uniform in the flow core region by considering that the pressure gradient in a cross section keeps balance with the body force. The assumed secondary flow streamlines are also shown in Fig. 1. Since the flow is fully developed, the boundary-layer thickness is invariable in the direction of pipe axis.

We use the cylindrical co-ordinates (r, ψ, Z) putting the line $\psi = 0$ on the secondary flow streamline passing through the center of a cross section. All quantities are denoted by the time-average values of turbulent flow. The velocity components in the direction of r, ψ and Z are U, V and W respectively, and these are made dimensionless by the radius of the pipe a and the kinematic viscosity ν as follows:

$$u = Ua/\nu, \qquad v = Va/\nu, \qquad w = Wa/\nu.$$

The dimensionless co-ordinates are defined as $\eta = r/a$, $z = Z/a$. The dimensionless pressure P is $P = (a^2/\nu^2)(p/\rho)$, where ρ is density. In a fully developed flow, P changes linearly with z. Thus,

$$\frac{\partial P}{\partial z} = -C \tag{1}$$

where C is a constant.

Under the condition of constant wall temperature gradient we may put the temperature T in the following form when the temperature distribution is fully developed.

$$T = \tau Z - G(r, \psi) \tag{2}$$

where τ is the constant wall temperature gradient along the pipe axis, and $G(r, \psi)$ is a function of r and ψ. The wall temperature T_w is set as $T_w = \tau Z$ by neglecting the temperature variation around the periphery of a cross section. This condition is realized with a rather thick pipe wall having good thermal conductivity. However, even when a little temperature variation really exists, the experiments on curved pipes [4] show that the Nusselt number based on the mean wall temperature around the periphery agrees with the result obtained under the present assumption. Dimensionless temperature is defined by

$$g = G/\tau a.$$

The heat fluxes in the r, ψ directions and at the wall are denoted by Q_r, Q_ψ, and Q_w respectively. Dimensionless heat fluxes are formed as follows:

$$q_\eta = Q_r/k\tau, \qquad q_\psi = Q_\psi/k\tau, \qquad q_w = Q_w/k\tau$$

where k is the thermal conductivity of the fluid.

Now we formulate the fundamental equations commonly applicable to various secondary flow problems.

The axial force balance equation:

$$\frac{\partial}{\eta\,\partial\eta}(\eta\tau_{z\eta}) + \frac{\partial\tau_{z\psi}}{\eta\,\partial\psi} = -C \tag{3}$$

where $\tau_{z\eta}$ and $\tau_{z\psi}$ are shear stresses expressed by

$$\tau_{z\eta} = \frac{\partial w}{\partial\eta} - uw - \overline{u'w'}$$
$$\tau_{z\psi} = \frac{\partial w}{\eta\,\partial\psi} - vw - \overline{v'w'}. \tag{4}$$

The first terms on the right-hand side of equation (4) are viscous stresses; the second terms shear stresses due to secondary flow; the last terms Reynolds stresses.

The equation of continuity:

$$\frac{\partial}{\eta\,\partial\eta}(\eta u) + \frac{\partial v}{\eta\,\partial\psi} = 0. \tag{5}$$

The energy balance equation:

$$\frac{\partial}{\eta\,\partial\eta}(\eta q_\eta) + \frac{\partial q_\psi}{\eta\,\partial\psi} = Pr\, w. \tag{6}$$

Heat fluxes are

$$\left.\begin{aligned} q_\eta &= -\frac{\partial g}{\partial\eta} + Pr\, ug + Pr\,\overline{u'g'} \\ q_\psi &= -\frac{\partial g}{\eta\,\partial\psi} + Pr\, vg + Pr\,\overline{v'g'} \end{aligned}\right\} \tag{7}$$

where $\overline{u'g'}$, $\overline{v'g'}$ are heat fluxes due to turbulent fluctuation. The conduction term in the direction of Z is neglected.

1.2. *The velocity and temperature distributions in the flow core region*

In the flow core region the stresses and the heat fluxes due to secondary flow are predominant, so we may put

$$\tau_{z\eta} = -u_1 w_1, \qquad \tau_{z\psi} = -v_1 w_1 \tag{8}$$

$$q_\eta = Pr\, u_1 g_1, \qquad q_\psi = Pr\, v_1 g_1 \tag{9}$$

where the suffix 1 denotes the value in the flow core region.

Substitution of equations (8) and (9) into equations (4) and (7) yields the following equations

$$u_1 \frac{\partial w_1}{\partial\eta} + v_1 \frac{\partial w_1}{\eta\,\partial\psi} = C \tag{10}$$

$$u_1 \frac{\partial g_1}{\partial\eta} + v_1 \frac{\partial g_1}{\eta\,\partial\psi} = w_1. \tag{11}$$

The particular solutions of equations (10–11) are obtained as follows:

$$\begin{aligned} u_1 &= D\cos\psi \\ v_1 &= -D\sin\psi \\ w_1 &= A + (C/D)\eta\cos\psi \end{aligned} \tag{12}$$

$$g_1 = A' + (C/2D^2)\,\eta^2\cos^2\psi + (A/D)\eta\cos\psi \tag{13}$$

where A, A' and D are constants. It is clear that u_1 and v_1 in equation (12) satisfy equation (5). The constant D represents the dimensionless velocity of uniform secondary flow in the flow core.

1.3. *The shear stress at the wall and the velocity distributions in the boundary layer*

We may assume, confining our attention to the region very near the wall, that the local law of friction is approximately the same as that observed in a flow where there is no secondary flow. According to the law of friction, the shear stress at the wall τ_w is expressed by the distance of a certain point from the wall and the velocity at that point. The relation between these quantities is derived from the resistance formula commonly used for a turbulent flow [4]. We define the resistance coefficient as

$$\lambda = \left(-\frac{\partial p}{\partial Z}\right)\frac{2a}{\frac{1}{2}\rho\, W_m^2} \tag{14}$$

where W_m is the mean velocity. The resistance coefficient for the case without secondary flow λ_0 is expressed in the following general form.

$$\lambda_0 = \alpha Re^{-1/m} \tag{15}$$

where Re is the Reynolds number defined by $Re = 2a\,W_m/\nu$, α is a constant coefficient and m is an integer, usually 4 or 5.

We denote the distance from the wall by ξ, and denote the values on the point where secondary flow component v has a maximum value with subscript f. From equation (15), the dimensionless shear stress at the wall $\check{w}^{*2}$ is expressed in terms of ξ_f and $\check{w}_f$.

$$\check{w}^{*2} \equiv \left(\frac{a}{\nu}\right)^2 \frac{\tau_w}{\rho} = \frac{\alpha}{2^{3+1/m}}\left\{\frac{(2m-1)^2}{m(4m-1)}\right\}^{(2m-1)/m} \times \check{w}_f^{(2m-1)/m}\,\xi_f^{-1/m} \tag{16}$$

where $\check{w}_f = \sqrt{(v_f^2 + w_f^2)}$.

One of the quantities determining the flow and temperature fields in the present analysis is the boundary-layer thickness which appears in the expression for the velocity and temperature distributions in the boundary layer. The boundary-layer thickness made dimensionless by the radius of a pipe a is denoted by δ. Corresponding to equation (15), the velocity distributions are written in the form following the $1/(2m-1)$-power law near the wall. The boundary condition for w is

$$\text{at} \quad \xi = \delta, \qquad w = w_{1\delta}$$

where $w_{1\delta}$ is the value of w_1 at $\xi = \delta$.

We write w as

$$w = w_{1\delta}\left(\frac{\xi}{\delta}\right)^{1/(2m-1)} \tag{17}$$

The circumferential (ψ) velocity component v is written in a form satisfying the boundary condition,

$$\text{at} \quad \xi = \delta, \qquad v = v_1$$

and the following condition

$$\int_0^\delta v \, d\xi = D(1-\delta)\sin\psi. \tag{18}$$

Equation (18) expresses the equality between the rate of secondary flow in the flow core and returning flow rate in the boundary layer [4]. We write v as

$$v = -D\sin\psi\left[-\frac{m}{m-1}\left(\frac{2}{\delta}-1\right)\left(\frac{\xi}{\delta}\right)^{1/(2m-1)} + \frac{1}{m-1}\left(m\frac{2}{\delta}-1\right)\left(\frac{\xi}{\delta}\right)\right]. \tag{19}$$

The components of shear stress at the wall in z and ψ directions are written as follows in consideration of $w \gg v$ [4].

$$w^{*2} = \frac{w_f}{\sqrt{(v_f^2 + w_f^2)}}\,\tilde{w}^{*2} \approx \tilde{w}^{*2}$$
$$= \frac{\alpha}{2^{3+1/m}}\left\{\frac{(2m-1)^2}{m(4m-1)}\right\}^{(2m-1)/m} \times w_{1\delta}^{(2m-1)/m}\,\delta^{-1/m} \tag{20}$$

$$v^{*2} = \frac{v_f}{\sqrt{(v_f^2 + w_f^2)}}\,\tilde{w}^{*2} \approx \frac{v_f}{w_f}\,\tilde{w}^{*2}$$
$$\approx \frac{\alpha m}{2^{1+1/m}(2m-1)}\left\{\frac{(2m-1)^2}{m(4m-1)}\right\}^{(2m-1)/m} \times w_{1\delta}^{(m-1)/m}\,\delta^{-(m+1)/m}\,D\sin\psi. \tag{21}$$

1.4. *The heat flux at the wall and the temperature distribution in the boundary layer*

We may assume that the heat-transfer mechanism in the region very near the pipe wall is approximately the same as that observed in the flow free from the influence of body force. Therefore, the heat flux at the wall is derived from the Nusselt number formula commonly used for a turbulent flow. The Nusselt number is defined as

$$Nu = \frac{2a\,Q_{wm}}{k(T_w - T_m)} \tag{22}$$

where Q_{wm} is the mean value of Q_w around the periphery of a cross section ($\psi = -\pi \sim \pi$), and T_m is the mixed mean temperature defined by

$$T_m = \frac{1}{\pi a^2 W_m}\int_{-\pi}^{\pi}\int_0^a W T r \, dr \, d\psi.$$

When the Nusselt number in the absence of secondary flow Nu_0 is given in the following form

$$Nu_0 = \beta\, Re^{(m-1)/m}\, Pr^{\kappa} \tag{23}$$

the heat flux at the wall is given by

$$q_w = \hat{q}_w Pr^{\kappa}\, w_{1\delta}^{(m-1)/m}\,\delta^{-1/m}\, g_{1\delta} \tag{24}$$

where β is the constant coefficient, κ the constant, $g_{1\delta}$ the value of g_1 at $\xi = \delta$ and

$$\hat{q}_w = 2^{-(m+1)/m}\left\{\frac{(2m-1)^2}{m(4m-1)}\right\}^{(m-1)/m}\frac{4m-1}{2m+1}\,\beta. \tag{25}$$

The derivation of equation (24) from equation (23) is shown in detail in [4]. The temperature distribution in the boundary layer is written as

$$g = g_1\left(\frac{\xi}{\delta}\right)^{1/n} \tag{26}$$

where n is an unknown quantity to be determined by the boundary-layer energy integral equation.

1.5. *Determination of A, C and A′*

In order to reduce the number of unknown quantities, A, C and A' are related to D and δ in the following manner. The boundary-layer thickness δ is actually a function of ψ. However, the variation of δ with ψ is considered to be negligibly small. Therefore, in the following analysis, δ is treated as constant representing the mean value taken along the periphery ($\psi = -\pi \sim \pi$).

The dimensionless mean velocity w_m is calculated by the following equation

$$w_m \equiv \frac{Re}{2} = \frac{1}{\pi}\left\{\int_{-\pi}^{\pi}\int_{0}^{1-\delta} w_1 \eta \, d\eta \, d\psi + \int_{-\pi}^{\pi}\int_{0}^{\delta} w(1-\xi)\, d\xi \, d\psi\right\}. \tag{27}$$

Substitution of equations (12) and (17) into equation (27) yields the following relation between A and δ.

$$A = \frac{Re}{2}\frac{1}{1 - \frac{1}{m}\delta + \frac{1}{4m-1}\delta^2}. \tag{28}$$

The equation describing the balance of forces exerted upon a portion of fluid bounded by the pipe wall and two cross sections a unit distance apart is

$$\int_{-\pi}^{\pi}\int_{0}^{a} \frac{\partial p}{\partial Z} r \, dr \, d\psi = \int_{-\pi}^{\pi} \tau_w a \, d\psi. \tag{29}$$

Use of the dimensionless quantities gives the following relation between C and w^{*2} averaged around the periphery.

$$C = 2w_m^{*2}. \tag{30}$$

In equation (20), $w_{1\delta}^{(2m-1)/m}$ is expanded as follows, assuming that $A \gg C/D$.

$$w_{1\delta}^{(2m-1)/m} = A^{(2m-1)/m} \times \left\{1 + \frac{2m-1}{m}\frac{C}{AD}\cos\psi + \ldots\right\}. \tag{31}$$

From equations (20, 28, 30, 31), the following relation between C and δ is obtained.

$$C = \frac{\alpha}{16}\left\{\frac{(2m-1)^2}{m(4m-1)}\right\}^{(2m-1)/m} Re^{(2m-1)/m} \times \delta^{-1/m}\left[1 + \frac{2m-1}{m^2}\delta\right]. \tag{32}$$

Here the terms having magnitudes of order less than δ^2 are neglected.

The equation describing the heat balance on a portion of fluid bounded by the pipe wall and two cross sections a unit distance apart is

$$\int_{-\pi}^{\pi} Q_w a \, d\psi = \rho c_p \frac{\partial}{\partial Z}\int_{-\pi}^{\pi}\int_{0}^{a} wT \, r \, dr \, d\psi. \tag{33}$$

This is expressed in the following dimensionless form:

$$q_{wm} = \frac{RePr}{4} \tag{34}$$

where q_{wm} is the mean value of q_w around the periphery of a cross section.

In equation (24), $w_{1\delta}^{(m-1)/m}$ is expanded as follows:

$$w_{1\delta}^{(m-1)/m} = A^{(m-1)/m} \times \left[1 + \frac{m-1}{m}\frac{C}{AD}\cos\psi + \ldots\right]. \tag{35}$$

From equations (24, 28, 34, 35), the following relation between A', D and δ is obtained;

$$A' = (Pr^{1-\kappa} - \Delta_{A'})\frac{Re^{1/m}\delta^{1/m}}{2^{1+1/m}\hat{q}_w\left(1 + \frac{m-1}{m^2}\delta\right)} \tag{36}$$

where

$$\Delta_{A'} = \frac{\alpha\beta}{16}\left\{\frac{(2m-1)^2}{m(4m-1)}\right\}^{(3m-2)/m} \times \frac{(4m-1)(3m-2)}{4m(2m+1)}\frac{Re^{2(m-1)/m}}{D^2\delta^{2/m}}. \tag{37}$$

Equations (28, 32, 36) reduce the unknown quantities to D, δ and n which appear in equation (26). These unknowns are determined by considering the momentum and heat balance in the boundary layer.

1.6. *The boundary-layer momentum 'ntegral equation in the axial* (Z) *direction*

The integral equation expressing the equilibrium of momentum in the axial (z) direction is

$$w^{*2} = w_{1\delta}\frac{\partial}{\partial\psi}\int_0^\delta v\,\mathrm{d}\xi - \frac{\partial}{\partial\psi}\int_0^\delta vw\,\mathrm{d}\xi + C\delta. \quad (38)$$

The way of obtaining a relation between D and δ from equation (38) is the same as that shown in [4]. The relation is

$$D^2\delta^{2/m} = \frac{\alpha^2(4m^2-1)(4m-1)}{2^7 m(6m-1)} \times \left\{\frac{(2m-1)^2}{m(4m-1)}\right\}^{2(2m-1)/m} Re^{2(m-1)/m}. \quad (39)$$

1.7. *The boundary-layer energy integral equation*

The energy integral equation is

$$\frac{q_w}{Pr} = g_{1\delta}\frac{\partial}{\partial\psi}\int_0^\delta v\,\mathrm{d}\xi - \frac{\partial}{\partial\psi}\int_0^\delta gv\,\mathrm{d}\xi + \int_0^\delta w\,\mathrm{d}\xi. \quad (40)$$

The parameter n is determined from equation (40) in the same way as that shown in [4]. In the present analysis, $g_{1\delta}$ is approximated in the following linearized form,

$$g_{1\delta} = A' + \frac{C}{4D^2} + \frac{A}{D}\cos\psi. \quad (41)$$

The result shows that with reasonable accuracy we may put

$$n = 2m - 1. \quad (42)$$

1.8. *Resistance coefficients and Nusselt numbers*

The definition of λ given by equation (14) is expressed in the following non-dimensional form

$$\lambda = \frac{16C}{Re^2}. \quad (43)$$

From equations (32) and (43), we obtain

$$\lambda = \alpha\left\{\frac{(2m-1)^2}{m(4m-1)}\right\}^{(2m-1)/m} \times \frac{1}{Re^{1/m}\delta^{1/m}}\left[1 + \frac{2m-1}{m^2}\delta\right]. \quad (44)$$

The mean heat flux in equation (22) is given by equation (34) in dimensionless form. Equation (22) is expressed in non-dimensional form as

$$Nu = \frac{RePr}{2g_m} \quad (45)$$

where

$$g_m = \frac{2}{\pi Re}\left[\int_{-\pi}^{\pi}\int_0^{1-\delta} g_1 w_1 \eta\,\mathrm{d}\eta\,\mathrm{d}\psi + \int_{-\pi}^{\pi}\int_0^{\delta} gw(1-\xi)\,\mathrm{d}\xi\,\mathrm{d}\psi\right]. \quad (46)$$

Equations (12, 13, 17, 26, 42) are substituted into equation (46). From equations (45) and (46), we obtain Nu, neglecting the terms less than order of magnitude δ^2 in the bracket, as

$$Nu = \frac{2^{1/m}\hat{q}_w Pr}{Pr^{1-\kappa} - \Delta g_m} Re^{(m-1)/m}\delta^{-1/m} \times \left[1 + \frac{4m^2-2m-1}{m^2(2m+1)}\delta\right] \quad (47)$$

where

$$\Delta g_m = \frac{m(3m-4)(4m-1)(6m-1)}{(2m+1)^2(2m-1)^3}\frac{\beta}{\alpha}. \quad (48)$$

When we find the relation between δ and the parameter inherent to the type of body force causing the secondary flow, we obtain λ and Nu for the problem being considered.

2. HEAT TRANSFER IN A STRAIGHT PIPE ROTATING ABOUT A PARALLEL AXIS

2.1. *Fundamental equations*

The particular relation between D and δ is given by the equation of force balance in the r- and ψ-directions. The diagram of the present problem is shown in Fig. 2. The pipe rotates with angular velocity ω, and the radius of rotation of the pipe axis is R. Due to the Coriolis force acting on the secondary flow, the line $\psi = 0$ does not necessarily lie on the line $\overline{0B}$ passing through the center of a pipe cross

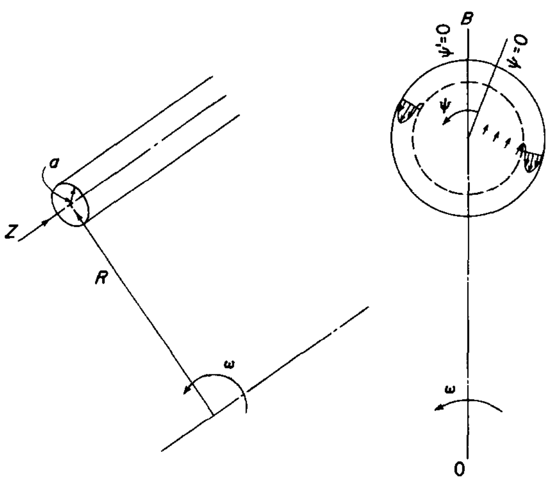

FIG. 2. A pipe rotating around a parallel axis.

section and the center of rotation. The angular co-ordinate measured from the line $\overline{0B}$ is denoted by ψ'. The deviation angle between ψ and ψ' is denoted by $\Delta(\equiv \psi - \psi')$ which is shown to be very small in the following analysis.

The acceleration terms in the direction of η and Ψ are

$$\text{acc}\,\eta = u\frac{\partial u}{\partial \eta} + \frac{v}{\eta}\frac{\partial u}{\partial \psi} - \frac{v^2}{\eta} - Jv - \frac{a^3}{\nu^2}R\omega^2\left(\cos\psi' + \frac{a\eta}{R}\right) \tag{49}$$

$$\text{acc}\,\psi = u\frac{\partial v}{\partial \eta} + \frac{v}{\eta}\frac{\partial v}{\partial \psi} + \frac{uv}{\eta} + Ju + \frac{a^3}{\nu^2}R\omega^2 \sin\psi' \tag{50}$$

where

$$J = 2a^2\omega/\nu.$$

The terms with parameter J express the Coriolis force due to secondary flow.

When the fluid and wall are kept in isothermal condition, pressure P_0 is caused by the centrifugal force.

$$P_0 = \frac{a^3}{\nu^2}R\omega^2\,\eta\left(\cos\psi' + \frac{a\eta}{2R}\right) + P_c \tag{51}$$

where P_c is the pressure at the center of the pipe.

When a pipe is heated or cooled, the temperature distribution in the fluid causes a secondary flow and a deviation of pressure from P_0. We denote the deviation of pressure by P, thus, $(P_0 + P)$ replaces P in the original equations whose inertia terms are expressed by equations (49) and (50). The density change due to temperature distribution introduces the following Grashof number in the dimensionless fundamental equations.

$$Gr = \frac{R\omega^2\gamma\,(\tau a)(2a)^3}{\nu^2}.$$

where γ is the coefficient of volumetric expansion of the fluid. It is noted that the gravitational acceleration in the definition of Grashof number for a natural convection problem is now replaced by a centrifugal acceleration $R\omega^2$.

2.2. *Pressure balance in the flow core*

In the core region, the pressure distribution is caused by the body force as follows:

$$\frac{\partial P_1}{\partial \eta} = -JD\sin\psi + \tfrac{1}{8}Grg_1\cos\psi' \tag{52}$$

$$\frac{\partial P_1}{\eta\,\partial \psi} = -JD\cos\psi - \tfrac{1}{8}Grg_1\sin\psi'. \tag{53}$$

From equations (52) and (53), we obtain

$$\cos\psi'\frac{\partial g_1}{\eta\,\partial\psi} + \sin\psi'\frac{\partial g_1}{\partial\eta} = 0. \tag{54}$$

The dimensionless temperature g_1 of equation (13) satisfies equation (54), when $\psi \approx \psi'$.

2.3. *The boundary-layer momentun integral equation in the circumferential* (ψ) *direction*

The integral equation is written as

$$v^{*2} = v_1 \frac{\partial}{\partial\psi}\int_0^\delta v\,\mathrm{d}\xi - \frac{\partial}{\partial\psi}\int_0^\delta v^2\,\mathrm{d}\xi - J\int_0^\delta u\,\mathrm{d}\xi - \delta\frac{\mathrm{d}P_{1\delta}}{\mathrm{d}\psi} - \tfrac{1}{8}Gr\int_0^\delta g\sin\psi'\,\mathrm{d}\xi \quad (55)$$

where $P_{1\delta}$ is the value of P_1 at $\xi = \delta$.

We average both sides of equation (55) on a semi-circle, $\psi = 0 \sim \pi$, to obtain the mean value of the boundary-layer thickness with reasonable accuracy [4]. We obtain the following equation denoting the averaged value by suffix m.

$$v_m^{*2} = -\delta\left(\frac{\mathrm{d}P_{1\delta}}{\mathrm{d}\psi}\right)_m - \tfrac{1}{8}Gr\left(\int_0^\delta g\sin\psi'\,\mathrm{d}\xi\right)_m. \quad (56)$$

From equation (53), we find

$$\left(\frac{\mathrm{d}P_{1\delta}}{\mathrm{d}\psi}\right)_m = -\tfrac{1}{8}Gr(g_{1\delta}\sin\psi')_m. \quad (57)$$

Substitution of equation (26) with equation (42) into equation (56) yields the following relation.

$$v_m^{*2} = \frac{1}{16m}Gr\delta(g_{1\delta}\sin\psi')_m. \quad (58)$$

The right-hand side of equation (58) is obtained in the final form by substitution of equation (41). We may assume $A' \gg C/4D^2$, considering that g_1 has a gentle gradient in the core region. Hence, we put in equation (41)

$$A' + \frac{C}{4D^2} \approx A' \approx \frac{Pr^{1-\kappa}}{2^{1+1/m}\hat{q}_w}Re^{1/m}\,\delta^{1/m}. \quad (59)$$

The left-hand side of equation (58) is given by averaging equation (21) with the aid of expansion shown in equation (35). When Δ is neglected ($\psi' \approx \psi$), the following relation between D and δ is obtained.

$$D\delta^{-2(m+1)/m} = Pr^{1-\kappa}\frac{(4m^2-1)}{4m^2(4m-1)} \times\left\{\frac{m(4m-1)}{(2m-1)^2}\right\}^{(3m-2)/m}\frac{1}{\alpha\beta}GrRe^{-(m-2)/m}. \quad (60)$$

From equations (39) and (60), D and δ are obtained as follows:

$$D = \hat{D}Re^{m/(m+1)}\Gamma^{-1/2(m+1)} \quad (61)$$

$$\log\hat{D} = \frac{1}{2m+3}[(m+2)\log(2m+1) + 9m\log(2m-1) - (5m+2)\log m - (3m-1)\log(4m-1) - (m+1)\log(6m-1) - (7m+9)\log 2 + (2m+1)\log\alpha - \log\beta] \quad (62)$$

$$\delta = \hat{\delta}\left(\frac{\Gamma^{m/2}}{Re}\right)^{1/(m+1)} \quad (63)$$

$$\log\hat{\delta} = \frac{1}{4m+6}[-m\log(2m+1) + (19m-12)\log(2m-1) - (7m-6)\log m - (7m-6)\log(4m-1) - m\log(6m-1) - 3m\log 2 + 4m\log\alpha + 2m\log\beta] \quad (64)$$

where

$$\Gamma = \left[\frac{Re^{2m+1}}{(GrPr^{1-\kappa})^{m+1}}\right]^{2/(2m+3)} \quad (65)$$

2.4. *Physical meaning of the parameter* Γ

The parameter Γ can be explained as

$$\Gamma = \frac{\text{inertia force}}{\text{body force}}$$

The inertia force is represented by Re^2. We denote the body force by X. The combinations of inertia and body force in different secondary flow problems give different Γ as shown in the following.

(1) In the present problem, the body force is represented by

$$X \sim GrA' \sim GrPr^{1-\kappa}Re^{1/m}\,\delta^{1/m}.$$

For turbulent flow, Γ is expressed by equation (65). For laminar flow, we put $m = 1$, $\kappa = 0$, and obtain

$$\Gamma = [Re^3/Ra^2]^{\frac{2}{5}}$$

where $Ra = GrPr$ (Rayleigh number).

(2) In a curved pipe, the centrifugal force caused by the mean velocity represents the body force.

$$X \sim W_m^2/R \sim Re^2(a/R)$$

where R is the radius of curvature of the pipe axis. Thus, we find $\Gamma = R/a$ (the radius ratio).

(3) In a pipe rotating around an axis perpendicular to its own axis [5], the body force causing secondary flow is the Coriolis force. We write X using a non-dimensional angular velocity $\hat{\omega} = 2a^2\omega/\nu$ as

$$X \sim \omega W_m \sim \hat{\omega} Re.$$

Hence, we find $\Gamma = Re/\hat{\omega}$.

we put $m = 4$. Though equation (66) is applicable in rather low Reynolds number region for an ordinary pipe flow, the formula obtained for curved pipes agrees well with the experimental results in the wide range of $Re/\Gamma^{m/2}$ [4].

In fluids having Prandtl numbers of about unity, i.e. most gases, the temperature distribution follows closely the law of velocity distribution. Hence, we use the value $m = 4$ in the heat transfer analysis for such fluids. As the basis to give the heat flux at the wall, we derived the following simplified Nu_0 from the formula for a turbulent flow in the rather low Reynolds number region [4].

$$Nu_0 = 0{\cdot}038\, Re^{\frac{3}{4}}\, Pr^{\frac{1}{3}}. \tag{67}$$

According to the fact that λ obtained by putting $m = 4$ covers a wide range of $Re/\Gamma^{m/2}$, the Nusselt number formula is also considered to cover a wide range of $Re/\Gamma^{m/2}$ when the fluids are gases. The formula obtained for curved pipes agrees well with the experimental results for air [4].

Table 1

Type of body force	$Re/\Gamma^{m/2}$		
	laminar	turbulent	
	$(m = 1)$	$m = 4$	$m = 5$
Density difference	$(RaRe)^{2/5}$	$Re^{-25/11}(GrPr^{\frac{2}{3}})^{20/11}$	$Re^{-42/13}(GrPr^{0{\cdot}6})^{30/13}$
Centrifugal force	$Re\,\sqrt{(a/R)}$	$Re(a/R)^2$	$Re(a/R)^{2{\cdot}5}$
Coriolis force	$(Re\,\hat{\omega})^{\frac{1}{2}}$	$\hat{\omega}^2/Re$	$\hat{\omega}^{2{\cdot}5}/Re^{1{\cdot}5}$

In all the secondary flow problems in a circular pipe, the resistance coefficient ratio λ/λ_0 and the Nusselt number ratio Nu/Nu_0 are expressed in terms of $Re/\Gamma^{m/2}$. The particular forms of $Re/\Gamma^{m/2}$ for the different problems are shown in Table 1.

When the turbulent shear stress at the wall is derived from the following Blasius's formula

$$\lambda_0 = 0{\cdot}316\, Re^{-\frac{1}{4}} \tag{66}$$

When we take the following formula

$$\lambda_0 = 0{\cdot}184\, Re^{-\frac{1}{5}} \tag{68}$$

we put $m = 5$. The resistance formula based on the friction law of equation (68) is applicable in a very high $Re/\Gamma^{m/2}$ region.

The formula of Nu_0 corresponding to equation (68) is Colburn's formula,

$$Nu_0 = 0{\cdot}023\, Re^{0{\cdot}8} Pr^{0{\cdot}4}. \tag{69}$$

Since equation (69) is supported by many empirical data of liquids, it is suggested that the formula based on equation (69) may be used for liquids in a practically important region of $Re/\Gamma^{m/2}$ [4].

2.5. *Resistance coefficients and Nusselt numbers*

The resistance coefficient of a rotating pipe is obtained by substitution of equation (63) into equation (44) as follows:

$$\lambda\sqrt{\Gamma} = \frac{\alpha_r}{\left(\dfrac{Re}{\Gamma^{m/2}}\right)^{1/(m+1)}}\left[1 + \frac{\Delta_\lambda}{\left(\dfrac{Re}{\Gamma^{m/2}}\right)^{1/(m+1)}}\right] \tag{70}$$

where

$$\alpha_r = \alpha\left\{\frac{(2m-1)^2}{m(4m-1)}\right\}^{(2m-1)/m} \hat{\delta}^{1/m} \tag{71}$$

$$\Delta_\lambda = \frac{2m-1}{m^2}\hat{\delta}. \tag{72}$$

It should be noted that, when $m = 4$, a little correction is necessary on the value of α in equation (15) to calculate the coefficient given by equation (71). The corrected value is $\alpha = 0{\cdot}305$, while the original value is $\alpha = 0{\cdot}316$ according to equation (66). This correction is made in order to make the coefficient of equation (16) with the widely recognized experimental result of τ_w [4]. We denote the original value of α by α_0 in the following expression for the resistance coefficient ratio λ/λ_0 to discriminate it from the corrected value.

$$\frac{\lambda}{\lambda_0} = \alpha_{r/0}\left(\frac{Re}{\Gamma^{m/2}}\right)^{1/m(m+1)} \left[1 + \frac{\Delta_\lambda}{\left(\dfrac{Re}{\Gamma^{m/2}}\right)^{1/(m+1)}}\right] \tag{73}$$

where

$$\alpha_{r/0} = \frac{\alpha}{\alpha_0}\left\{\frac{(2m-1)^2}{m(4m-1)}\right\}^{(2m-1)/m} \hat{\delta}^{-1/m}. \tag{74}$$

The Nusselt number is obtained by substituting equation (63) into equation (47).

$$Nu = \frac{\hat{N}uPr}{Pr^{1-\kappa} - \Delta g_m}\frac{Re^{m/(m+1)}}{\Gamma^{1/2(m+1)}} \times \left[1 + \frac{\Delta_{Nu}}{\left(\dfrac{Re}{\Gamma^{m/2}}\right)^{1/(m+1)}}\right] \tag{75}$$

where

$$\hat{N}u = 2^{1/m}\hat{q}_w\hat{\delta}^{-1/m} \tag{76}$$

$$\Delta_{Nu} = \frac{4m^2 - 2m - 1}{m^2(2m+1)}\hat{\delta}. \tag{77}$$

The Nusselt number ratio becomes

$$\frac{Nu}{Nu_0} = \frac{\hat{N}u}{\beta}\frac{Pr^{1-\kappa}}{Pr^{1-\kappa} - \Delta g_m}\left(\frac{Re}{\Gamma^{m/2}}\right)^{1/m(m+1)} \times \left[1 + \frac{\Delta_{Nu}}{\left(\dfrac{Re}{\Gamma^{m/2}}\right)^{1/(m+1)}}\right]. \tag{78}$$

We use the suffixes 4 and 5 to denote the value of m used in the calculation. When we set $m = 4$, from equations (66) and (67) we find

$\alpha = 0{\cdot}305$ (the corrected value), $\kappa = \frac{1}{3}$, $\beta = 0{\cdot}038$.

From equation (70), we obtain

$$\lambda_4\sqrt{\Gamma} = \frac{0{\cdot}338}{\left(\dfrac{Re}{\Gamma^2}\right)^{\frac{1}{5}}}\left[1 + \frac{0{\cdot}07}{\left(\dfrac{Re}{\Gamma^2}\right)^{\frac{1}{5}}}\right]. \tag{79}$$

The ratio λ_4/λ_0 is shown in Fig. 3. The curve in Fig. 3 may be used to estimate the increase in the flow resistance in a wide range of Re/Γ^2.

From equation (75), we obtain for gases ($Pr \approx 1$)

$$Nu_4 = \frac{0{\cdot}043\, Pr}{Pr^{\frac{2}{3}} - 0{\cdot}050}\frac{Re^{\frac{4}{5}}}{\Gamma^{\frac{1}{10}}}\left[1 + \frac{0{\cdot}061}{\left(\dfrac{Re}{\Gamma^2}\right)^{\frac{1}{5}}}\right] \tag{80}$$

The ratio Nu_4/Nu_0 is shown in Fig. 4. From equations (68) and (69), we find

$$m = 5, \quad \alpha = 0{\cdot}184, \quad \beta = 0{\cdot}023, \quad \kappa = 0{\cdot}4.$$

The resistance coefficient is written as

$$\lambda_5\sqrt{\Gamma} = \frac{0{\cdot}220}{\left(\dfrac{Re}{\Gamma^{2\cdot5}}\right)^{\frac{1}{6}}}\left[1 + \frac{0{\cdot}035}{\left(\dfrac{Re}{\Gamma^{2\cdot5}}\right)^{\frac{1}{6}}}\right]. \tag{81}$$

The ratio λ_5/λ_0 which is expected to be valid for very large $Re/\Gamma^{2\cdot5}$ is shown in Fig. 5.

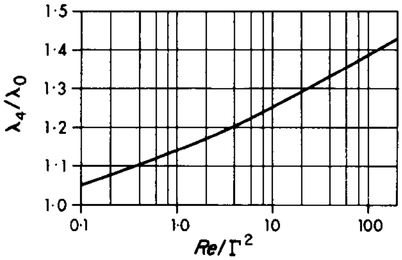

FIG. 3. Resistance coefficient ratio (λ_4/λ_0) vs. Re/Γ^2.

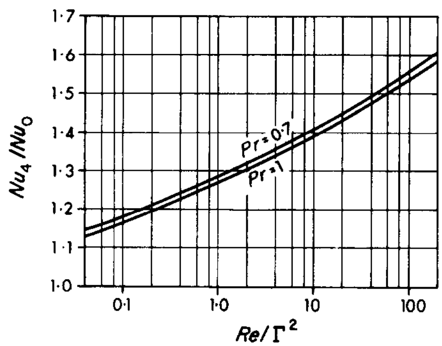

FIG. 4. Nusselt number ratio for gases.

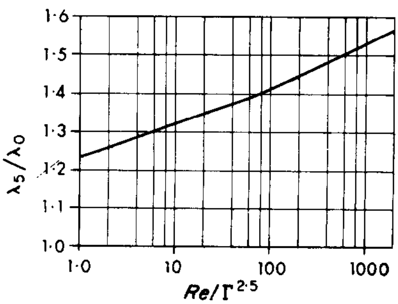

FIG. 5. Resistance coefficient ratio (λ_5/λ_0) vs. $Re/\Gamma^{2\cdot5}$.

For Prandtl numbers larger than unity, in equation (75), we may put

$$\frac{Pr^{1-\kappa}}{Pr^{1-\kappa} - \Delta g_m} \approx 1.$$

Hence, we obtain the Nusselt number for liquids as

$$Nu_5\, Pr^{-0\cdot4} = 0{\cdot}028\,\frac{Re^{\frac{5}{6}}}{\Gamma^{\frac{1}{12}}}\left[1 + \frac{0{\cdot}032}{\left(\dfrac{Re}{\Gamma^{2\cdot5}}\right)^{\frac{1}{6}}}\right] \tag{82}$$

The ratio Nu_5/Nu_0 is shown in Fig. 6.

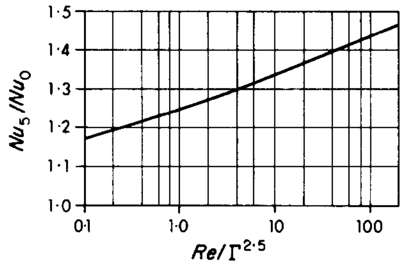

FIG. 6. Nusselt number ratio for liquids.

Figures 3, 4, 5 and 6 show that the increases in the flow resistance and the heat-transfer rate in a turbulent flow are not so remarkable as those found in a laminar flow [3].

The expressions of D and δ are given as follows:

$(m = 4)$ $$\delta_4 = 0{\cdot}160\,(Re/\Gamma^2)^{-\frac{1}{5}} \tag{83}$$

$$D_4 = 0{\cdot}096\, Re^{\frac{4}{5}}\, \Gamma^{-\frac{1}{10}} \tag{84}$$

$(m = 5)$ $$\delta_5 = 0{\cdot}098\,(Re/\Gamma^{2\cdot5})^{-\frac{1}{6}} \tag{85}$$

$$D_5 = 0{\cdot}070\, Re^{\frac{5}{6}}\, \Gamma^{-\frac{1}{12}}. \tag{86}$$

2.6. *The effect of Coriolis force*

In order to obtain $\Delta(= \psi - \psi')$, in equation (53) we put

$$\eta = 1, \quad \psi = 0, \quad \partial P_1/\partial\psi = 0, \quad \psi' = -\Delta.$$

From equation (53), we obtain

$$\sin\Delta = \frac{8JD}{Gr\left(A' + \dfrac{C}{4D^2} + \dfrac{A}{D}\right)}. \tag{87}$$

The angle Δ is supposed to be small, therefore, we set $\sin\Delta \approx \Delta$. We give the suffix 0 to the quantities obtained in the previous sections to denote the zero-th order approximation for the Coriolis effect. Equation (87) is written as

$$\Delta = \frac{8\hat{D}_0}{A_0' + 1/(2\hat{D}_0 \mathrm{Pr}^{1-\kappa})} M \tag{88}$$

where

$$M = JRe^{(2m-5)/(2m+3)} (GrPr^{1-\kappa})^{-(2m+1)/(2m+3)} \tag{89}$$

$$\hat{A}_0' = \delta_0^{1/m}/2^{1+1/m}\hat{q}_w. \tag{90}$$

The physical meaning of M is given by

$$M = \frac{\text{Coriolis force due to secondary flow}}{\text{body force}}.$$

By putting $\cos\Delta \approx 1$ and $\sin\Delta \approx \Delta$, we find

$$(g_{1\delta}\sin\psi')_m = \left(A' + \frac{C}{4D^2}\right)\frac{2}{\pi} - \frac{A}{2D}\Delta. \tag{91}$$

Equation (91) is substituted into equation (58), and D is eliminated by using equation (39). Thus, the equation for δ is obtained. The expansion is made as

$$\delta = \delta_0(1 + \Delta_\delta + \ldots). \tag{92}$$

After eliminating negligibly small terms, Δ_δ is obtained as follows from equation (58).

$$\Delta_\delta = \frac{\pi m \Delta}{4(2m+3)} \sqrt{\left[\frac{2m(6m-1)}{(4m^2-1)(4m-1)}\right]}. \tag{93}$$

The correction coefficient for λ and Nu is obtained in the following form.

$$\zeta = 1 - \frac{1}{m}\Delta_\delta. \tag{94}$$

The coefficient ζ is multiplied to the right-hand side of equations (70, 73, 75, 78), when the effects of Coriolis force is taken into account. However, the value of ζ is usually very close to unity as shown later. Thus, we may neglect the effect of Coriolis force in many cases.

When we put $m = 4$, equation (88) becomes

$$\Delta_4 = \frac{0{\cdot}147}{2{\cdot}23 + Pr^{-\frac{2}{3}}} M \tag{95}$$

where

$$M = JRe^{3/11}(GrPr^{\frac{2}{3}})^{-9/11}. \tag{96}$$

In case of $m = 5$,

$$\Delta_5 = \frac{0{\cdot}078}{2{\cdot}51 + Pr^{-0{\cdot}6}} M \tag{97}$$

where

$$M = JRe^{5/13}(GrPr^{0{\cdot}6})^{-11/13}. \tag{98}$$

The correction coefficient for $\lambda_4\sqrt{\Gamma}$, λ_4/λ_0, Nu_4 and Nu_4/Nu_0 is given by

$$\zeta_4 = 1 - \frac{0{\cdot}0046}{2{\cdot}23 + Pr^{-\frac{2}{3}}} M. \tag{99}$$

The correction coefficient for $\lambda_5\sqrt{\Gamma}$, λ_5/λ_0, $Nu_5 Pr^{-0{\cdot}4}$ and Nu_5/Nu_0 is given by

$$\zeta_5 = 1 - \frac{0{\cdot}002}{2{\cdot}51 + Pr^{-0{\cdot}6}} M. \tag{100}$$

Let us estimate ζ for the following example. The data are given as follows:

diameter of a pipe $2a = 10$ mm,
radius of rotation $R = 50$ cm,
angular velocity $\omega = 523$ rad/s (5000 rev/min),
fluid is air (the mixed mean temperature is around 40°C),
temperature gradient along the pipe axis $\tau = 20$°C/m, mean velocity $W_m = 20$ m/s.
The dimensionless parameters are found to be

$$Re = 1{\cdot}142 \times 10^4, Gr = 1{\cdot}53 \times 10^5, \Gamma = 1{\cdot}05 \times 10^2,$$

$$Re/\Gamma^2 = 1{\cdot}04, J = 1{\cdot}49 \times 10^3, M = 1{\cdot}33.$$

The increase in Nusselt number is given by $Nu/Nu_0 = 1{\cdot}282$. From equation (95), we find

$$\Delta_4 = 0{\cdot}056 \text{ rad} = 3{\cdot}2 \text{ deg}.$$

The correction coefficient for $\lambda_4\sqrt{\Gamma}$, λ_4/λ_0, Nu_4 and Nu_4/Nu_0 is

$$\zeta_4 = 0{\cdot}998.$$

The results shows only 0·2 per cent reduction. The Coriolis force exerts little influence on λ and Nu in turbulent flow, while a little influence is expected in laminar flow [3].

2.7. *Further remarks*

(1) In order to see the influence of the different factors explicitly, $Re/\Gamma^{m/2}$ is disintegrated into the following form.
For gases ($Pr \approx 1$),

$$Re/\Gamma^2 \sim R^{20/11} W_m^{-25/11} \omega^{40/11} \tau^{20/11} \times \nu^{-15/11}.$$

For liquids,

$$Re/\Gamma^{2\cdot 5} \sim R^{30/13} W_m^{-42/13} \omega^{60/13} \tau^{30/13} \times \nu^{-18/13} Pr^{18/13}.$$

In both cases, the effect of secondary flow increase remarkably with increase in angular velocity, and decreases with increasing mean velocity.

(2) When $R\omega^2$ in the definition of Gr is replaced by a gravitational acceleration (9·8 m/s^2), the calculation up to equation (86) may be applied to the problem of turbulent heat transfer in a horizontal pipe. The experimental data in a horizontal pipe are found in the paper by Mori *et al.* [6]. The experiment was done by using air. One of the data points, for example, is taken at

$$Re = 1{\cdot}2 \times 10^4$$

$$Gr = 440.$$

The value of Gr is calculated from $Ra = 38{\cdot}5$ found in [6] which is based on the radius of the pipe a. These values give $\Gamma = 2{\cdot}32 \times 10^4$ and $Re/\Gamma^2 = 2{\cdot}23 \times 10^{-5}$. It is clear that the effect of buoyancy is negligibly small. The velocity and temperature distributions obtained by experiment show little deviation from the symmetric turbulent flow and temperature profiles. The Nusselt number agreed with the formula for a symmetrical turbulent pipe flow.

CONCLUSIONS

Turbulent heat transfer in a pipe rotating around a parallel axis is studied theoretically on the assumption that flow and temperature fields are fully developed under the condition of constant wall temperature gradient. The following conclusive remarks are obtained.

(1) A secondary flow is expected due to the density difference in a centrifugal field when a temperature distribution exists in a pipe. Velocity and temperature distribution distorted by a sufficiently large secondary flow can be analysed by the boundary-layer approximation near the wall. The axial momentum balance equation and the energy balance equation are common to all the similar secondary flow problems caused by different kinds of body force. The particular equations for the present problem are the force balance equations in a cross section.

(2) The resistance coefficient λ and the Nusselt number Nu are expressed by use of an integer m which forms the exponents of Re in the formulae of $\lambda_0[\sim Re^{-1/m}]$ and $Nu_0[\sim Re^{(m-1)/m}]$. The formulae for λ and Nu are obtained by setting $m = 4$ or 5.

(3) The ratios λ/λ_0 and Nu/Nu_0 showing the increase in λ and Nu due to secondary flow are obtained in terms of $Re/\Gamma^{m/2}$, where Γ is the ratio of the inertia force to the body force. It is found that various parameters appearing in different secondary flow problems can generally be expressed by $Re/\Gamma^{m/2}$ in either laminar region ($m = 1$) or turbulent region.

The ratio λ/λ_0 obtained by setting $m = 4$ is considered to cover a wide range of Re/Γ^2. The ratios Nu/Nu_0 are obtained in terms of Re/Γ^2 for gases ($Pr \approx 1$) and $Re/\Gamma^{2\cdot 5}$ for liquids ($Pr > 1$). These results, though experimental confirmation is necessary, are available in the region of high rotational speeds where practical interests lie. The increases in λ and Nu in turbulent flow are not so remarkable as those encountered in laminar flow [3].

(4) The effect of Coriolis force acting on a flow is expressed in terms of M which represents the ratio of the Coriolis force to the body force. However, this effect in turbulent flow is so small that we may neglect it in many cases.

(5) The effect of secondary flow increases markedly with increase in the angular velocity, if we keep the other factors constant. The mean velocity has an inverse effect on λ/λ_0 and Nu/Nu_0 in turbulent flow, while these ratios increase positively with the mean velocity in laminar flow [3].

(6) The results obtained by the present analysis can be applied to a uniformly heated horizontal pipe if we replace the centrifugal acceleration in Gr by the gravitational acceleration. However, the effect of buoyancy in turbulent flow is usually negligible as Re/Γ^2 is very small. The experimental data [6] show little influence of the secondary flow effect.

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Résumé—Le transport de chaleur turbulent dans un tuyau tournant autour d'un axe parallèle avec une grande vitesse angulaire est étudié théoriquement en supposant un écoulement secondaire effectif dû à la différence de densité dans un champ centrifuge. Les champs d'écoulement et de température, entièrement établis sous la condition d'un gradient de température pariétale constante, sont analysés en les divisant en une région d'écoulement central et une couche limite mince le long de la paroi. L'analyse consiste de deux parties dans cet article. L'une est commune à différents problèmes d'écoulement secondaire, et l'autre dans laquelle les termes de force volumique apparaissent explicitement est particulière au problème actuel. Les formules pour le coefficient de résistance λ et le nombre de Nusselt Nu sont obtenues en fonction de Re et de Γ(Re = nombre de Reynolds, Γ = force d'inertie/force volumique). On montre que les augmentations des valeurs de λ et du Nu dues à l'écoulement secondaire peuvent être exprimées en fonction du paramètre $Re/\Gamma^{m/2}$ ($m = 1$ pour l'écoulement laminaire, $m = 4$ ou 5 pour l'écoulement turbulent) que l'on trouve être un paramètre général pour différents problèmes d'écoulement secondaires. L'effet de la force de Coriolis dû à l'écoulement secondaire est négligeable.

Zusammenfassung—Der turbulente Wärmeübergang in einem Rohr, das um eine parallele Achse mit grosser Winkelgeschwindigkeit rotiert, wird theoretisch untersucht, wobei eine wirksame Zweitströmung infolge von Dichteunterschieden in einem Zentrifugalfeld angenommen wird.

Geschwindigkeits- und Temperaturfelder, die unter der Bedingung konstanter Wandtemperaturgradient en voll ausgebildet sind, werden nach ihrer Aufteilung in einem Strömungskernbereich und eine dünne Grenzschicht an der Wand analysiert. Diese Analyse besteht aus zwei Teilen. Einer ist gebräuchlich für verschiedene sekundäre Strömungsprobleme und der andere, in dem die Ausdrücke für die Massenkraft explizit auftreten, ist eigentümlich für das vorliegende Problem. Die Gleichungen für den Widerstandskoeffizienten λ und die Nusselt-Zahl Nu lösen sich in Ausdrücken von Re und (Re—Reynolds-Zahl Γ = Trägheitskraft/Massenkraft.)

Es zeigt sich, dass das Anwachsen der Werte von λ und Nu auf Grund der Sekundärströmung in Form des Parameters $Re/\Gamma^{m/2}$ ($m = 1$ für Laminarströmung, $m = 4$ oder 5 für turbulente Strömung) ausgedrückt werden kann. Dieser Parameter erweist sich als allgemein gültig für verschiedene Sekundärströmungsprobleme. Der Einfluss der Cariolis–Kraft infolge der Sekundärströmung zeigt sich als vernachlässigbar klein.

Аннотация—Теоретически изучается турбулентный перенос в трубе, вращающейся вокруг параллельной оси с большой угловой скоростью пугем рассмотрения эффективного вторичного иотока, возникающего благодаря разности плотностей в центробежном поле. Исследуются поток и температурные поля, полностью развитые при постоянном градиенте температуры на стенке, путем разделения их на области ядра потока и тонкого пограничного слоя вдоль стенки. В данной статье анализ состоит из двух частей. Одна относится к различным проблемам вторичного течения, а вторая, в которой члены, описывающие силы, действующие со стороны тела, появляются в явном виде, характерна для данной задачи. Формула для коэффициента сопротивления λ и число Нуссельта Nu получены в числах Re и Γ (Re=число Рейнольдса, Γ=сила инерции/сила тепла). Показано что увеличение величины λ и Nu благодаря вторичному потоку может быть выражено параметром $Re/\Gamma^{m/2}$($m=1$ для ламинарного потока и $m=4$ или 5 для турбулентного потока), который является общим параметром для различных задач с вторичными течениями. Показано также, что эффект силы Кориолиса благодаря вторичному потоку пренебрежимо мал.